

## MIXED MINEFIELD MODELING

by

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The purpose of this paper is to present methodology to assess the effectiveness of Anti-Materiel (AM) minefield composed of a mixture of different types of AM fuze mines. In particular, the family of scatterable mines is of main concern. In order to minimize countermeasures and obtain an effective minefield it is necessary to develop mines with specialized functions and seed the minefield with an appropriate mixture, each type to perform its particular task. The following two questions are resolved: (1) What should be the optimum fuze mixture, (2) How sensitive is the optimum mixture to minefield parameters?

To accomplish this task, the paper is partitioned into two parts: Part I - A Minefield Plow Effectiveness Model, Part II - Optimum Anti-Materiel Minefield Fuzing. The first part is concerned with a model for assessing the effects of mine clearing plows. The second part employs the results of the first part and obtains the optimum fuze mixture in the presence of several possible enemy countermeasure strategies.

The effectiveness criteria employed is to minimize the target survival probability. Though a number of other effectiveness criteria can be readily defined, it is necessary that the minefield be credible - that is, it must present a significant threat to a target attempting a breach. Hence choosing the mixture that maximizes the target kill probability appears to be a reasonable approach.

The approach taken is to introduce a suitable collection of system states in order to represent the interaction between the components of the target array as a Markov process. Solution of the resulting coupled system of first order linear differential equations gives rise to survival probabilities for the target components as a function of the minefield parameters.

## PART I - A MINEFIELD PLOW EFFECTIVENESS MODEL

### INTRODUCTION

→ The purpose of this section is to present a model for analyzing the effectiveness of mine clearing plows mounted in front of the tracks of a tank in a AM minefield. The plow's function is to sweep AM mines away from the path of the tank's tracks thereby preventing a track - AM mine contact, thus increasing the tank's survivability.

→ The minefield to be considered consists of a mixture of AM munitions with three different types of fuzes; anti-handling (AH), pressure (PR), long impulse (LI). AH munitions will almost certainly be detonated upon contact with the plow whereas PR and LI munitions will usually be pushed aside without detonation. A major purpose of employing AH munitions in the minefield is to countermeasure plows.

### DEFINITIONS AND ASSUMPTIONS

The following definitions and assumptions are employed by the model:

1. The model considers a single tank with separate plows mounted in front of each track. The plows can be raised and lowered as required independently of each other. For example, if the left plow should become unusable, it can be raised. The tank can then proceed through the minefield using the right plow to clear mines from the path of the right track but with the left track vulnerable. If both plows should become damaged, they can both be raised, and the tank can then proceed without any plowing capability.

2. The plow can be envisioned as consisting of two parts: the moldboard and skidshoe. The mold board is used to do the plowing while the skidshoe is used to maintain the proper relationship between the moldboard and the ground.

Let:  $KMB$  denote the effective width of the moldboard  
 $KSH$  denote the effective width of the skidshoe

Thus:  $KMB \cdot \Delta x$  = the area of the minefield contacted by the mold board when the tank plow assembly moves distance  $\Delta x$  through the minefield.

$KSH \cdot \Delta x$  = the area of the minefield contacted by the skidshoe when the tank plow assembly moves distance  $\Delta x$  through the minefield.

3. The munitions employed are only effective only against the tank tracks. We denote the effective width of each track by  $KT$ , thus

$KT \cdot \Delta x$  = the area of the minefield contacted by one of the tank tracks when the tank plow assembly moves distance  $\Delta x$  through the minefield.

4. The minefield consists of a mixture of three types of munitions: AH, PR, AND LI. We define the minefield parameters  $\rho$ ,  $\lambda_1$ ,  $\lambda_2$ .

Where:

$\rho$  denotes the total minefield area density (mines/ft<sup>2</sup>)  
 $\lambda_1 \rho$  denotes the density of the AH mines (mines/ft<sup>2</sup>)  
 $\lambda_2 \rho$  denotes the density of PR (mines/ft<sup>2</sup>)  
 $(1-\lambda_1-\lambda_2)\rho$  denotes the density of LI mines (mines/ft<sup>2</sup>)

Note that  $\lambda_1, \lambda_2 \leq 0$ , and  $\lambda_1 + \lambda_2 \leq 1$

5a. When a plow contacts a mine we define the following mine detonation probabilities

<u>Mine type</u>	<u>Probability of Detonation</u>	
	<u>Moldboard</u>	<u>Skidshoe</u>
AH	PD1	PD2
PR	PD3	PD4
LI	PD5	PD6

Ideally, PD1 and PD2 = 1, PD5 and PD6 = 0

b. When a tank **track** contacts a mine we define the following mine detonation probabilities

<u>Mine type</u>	<u>Probability of Detonation</u>
AH	TD1
PR	TD2
LI	TD3

6a. When a mine detonates against a plow, we define the following plow kill probabilities

<u>Mine Type</u>	<u>Probability of Plow Kill</u>	
	<u>Moldboard</u>	<u>Skidshoe</u>
AH	PK1	PK2
PR	PK3	PK4
LI	PK5	PK6

b. When a mine detonates against a track, we define the following tank mobility kill probabilities

<u>Mine Type</u>	<u>Probability of Tank Mobility Kill</u>
AH	TK1
PR	TK1
LI	TK3

7. When a plow clears a mine, given that the mine does not detonate, there exists a small but non-zero probability that the mine can roll back into the path of the tank's track. We denote this probability for each munition as follows:

<u>Mine Type</u>	<u>Roll Back Probability/No Detonation</u>	
	<u>Moldboard</u>	<u>Skidshoe</u>
AH	PR1	PR2
PR	PR3	PR4
LI	PR5	PR6

### DIFFERENTIAL KILL PROBABILITIES

With the preceeding definitions and nomenclature, we are now in a position to obtain expressions for the differential tank and plow kill probabilities when the tank moves from  $x$  to  $x + \Delta x$ .

#### 1. Plow Kill

In order to determine the differential plow kill probability, we must consider the moldboard and skidshoe separately.

##### a. Moldboard:

In order for a moldboard plow kill to occur, the moldboard must contact a munition, the munition must detonate, and the detonation must inflict sufficient damage to the moldboard. One obtains considering all three munition types:

$$\begin{aligned} &\text{Prob (Moldboard plow kill in } x \text{ to } x + \Delta x) \\ &= PK_1 PD_1 \lambda_1 \rho K_{MB} \Delta x + PK_3 PD_3 \lambda_2 \rho K_{MB} \Delta x + PK_5 PD_5 (1 - \lambda_1 - \lambda_2) \rho K_{MB} \Delta x \\ &\triangleq \alpha_1 \rho \Delta x \end{aligned}$$

##### b. Skidshoe

Similarly for a skidshoe plow kill to occur, the skid shoe must contact a munition, the munition must detonate, and the detonation must inflict sufficient damage on the skid shoe to render it inoperative. One obtains considering all three munition types:

$$\begin{aligned} &\text{Prob (skidshoe plow kill in } x \text{ to } x + \Delta x) \\ &= PK_2 PD_2 \lambda_1 \rho K_{SH} \Delta x + PK_4 PD_4 \lambda_2 \rho K_{SH} \Delta x + PK_6 PD_6 (1 - \lambda_1 - \lambda_2) \rho K_{SH} \Delta x \\ &\triangleq \alpha_2 \rho \Delta x \end{aligned}$$

#### 2. Tank Track Kill

In order to obtain expressions for the differential tank track kill probability we must consider separately the case where the plow is down and the plow is raised

##### a. plow is raised:

As done for calculating plow kill probability, when the

plow is raised the differential track kill probability is given as the product of the mine contact probability, the mine detonation probability, and the kill given detonation probability. Consider all three munition types one obtains:

$$\begin{aligned} &\text{Prob (track kill in } x \text{ to } x + \Delta x / \text{plow raised)} \\ &= TK_1 TD_1 \lambda_1 \rho K_T \Delta x + TK_2 TD_2 \lambda_2 \rho K_T \Delta x + TK_3 TD_3 (1 - \lambda_1 - \lambda_2) \rho K_T \Delta x \\ &\triangleq a_3 \rho \Delta x \end{aligned}$$

b. plow is lowered:

When the plow is lowered in order for a track kill to occur, the mine must contact the plow, not detonate, roll back into the path of the tank track, detonate on the track, and inflict sufficient damage to the tank track to cause a tank mobility kill. One obtains considering the skid-shoe and skidshoe separately and all three munition types.

$$\begin{aligned} &\text{Prob (track kill in } x \text{ to } x + \Delta x / \text{plow lowered)} \\ &= TK_1 TD_1 PR_1 (1 - PD_1) \lambda_1 \rho K_{MB} \Delta x + TK_2 TD_2 PR_3 (1 - PD_3) \lambda_2 \rho K_{MB} \Delta x \\ &\quad + TK_3 TD_3 PR_5 (1 - PD_5) (1 - \lambda_1 - \lambda_2) \rho K_{MB} \Delta x \\ &\quad + TK_1 TD_1 PR_2 (1 - PD_2) \lambda_1 \rho K_{SH} \Delta x + TK_2 TD_2 PR_4 (1 - PD_4) \lambda_2 \rho K_{SH} \Delta x \\ &\quad + TK_3 TD_3 PR_6 (1 - PD_6) (1 - \lambda_1 - \lambda_2) \rho K_{SH} \Delta x \\ &= a_4 \rho \Delta x \end{aligned}$$

It should be observed that  $K_T$ , the effective track width, did not enter into this expression. The effective track width is built into the roll-back probabilities.

#### SYSTEM STATES AND STATE TRANSITION DIAGRAM

The survival probability of the tank and the plow are coupled since as long as the plow is functional it offers protection for the tank tracks. One can represent this complete system by defining the following distinct system states

$$S_{ij} = (i, j)$$

where  $i$  denotes the number of functional plows,  $= 0, 1$  or  $2$ , and  $j = 0$  or  $1$ , depending on whether the tank has suffered a mobility kill or not. We therefore have the following states.

- $S_1 = (2, 1)$ : both plows are functional and the tank has not suffered a mobility kill
- $S_2 = (1, 1)$ : one of the plows are functional and the tank has not suffered a mobility kill
- $S_3 = (0, 1)$ : both plows have been damaged but the tank has not suffered a mobility kill
- $S_{4a} = (2, 0)$ : both plows are functional but the tank has suffered a mobility kill
- $S_{4b} = (1, 0)$ : one of the plows are functional and the tank has suffered a mobility kill
- $S_{4c} = (0, 0)$ : both plows have been damaged and the tank has suffered a mobility kill

States  $S_{4a}$ ,  $S_{4b}$ , and  $S_{4c}$  represent the states for which a tank has suffered a mobility kill. Transitions between these states are of no relevance. Indeed, the tank cannot move and transitions cannot even occur. For most computational purposes they can be lumped into a collective state  $S_4$ , i.e.

$$S_4 = S_{4a} \cup S_{4b} \cup S_{4c}: \text{ the tank has suffered a mobility kill}$$

Using the definition of the system states and the differential kill probabilities derived in the preceding section one can construct the state transition diagram shown in Figure 1. By construction, the state transition diagram graphically portrays the differential probabilities of the system changing from one system state to another system state when the system moves from  $x$  to  $x + \Delta x$ .

#### SYSTEM DIFFERENTIAL EQUATION

From the state transition diagram, one immediately obtains the following set of coupled difference equations for the system states

$$\begin{bmatrix} P_{S1}(x+\Delta x) \\ P_{S2}(x+\Delta x) \\ P_{S3}(x+\Delta x) \\ P_{S4}(x+\Delta x) \end{bmatrix} = \begin{bmatrix} 1-2(a_1+a_2)\rho\Delta x & 0 & 0 & 0 \\ -2a_4\rho\Delta x & 1-(a_1+a_2)\rho\Delta x & 0 & 0 \\ 0 & -(a_3+a_4)\rho\Delta x & 1-2a_3\rho\Delta x & 0 \\ 2a_4\rho\Delta x & (a_3+a_4)\rho\Delta x & 2a_3\rho\Delta x & 1 \end{bmatrix} \begin{bmatrix} P_{S1}(x) \\ P_{S2}(x) \\ P_{S3}(x) \\ P_{S4}(x) \end{bmatrix}$$

$$\begin{aligned} P_{S4a}(x+\Delta x) &= P_{S1}(x)2a_4\rho\Delta x + P_{S4a}(x) \\ P_{S4b}(x+\Delta x) &= P_{S2}(x)(a_3+a_4)\rho\Delta x + P_{S4b}(x) \\ P_{S4c}(x+\Delta x) &= P_{S3}(x)2a_3\rho\Delta x + P_{S4c}(x) \end{aligned}$$

Taking the limit as  $\Delta x \rightarrow 0$ , the following system of differential equations results:

$$\begin{bmatrix} P_{S1}'(x) \\ P_{S2}'(x) \\ P_{S3}'(x) \\ P_{S4}'(x) \end{bmatrix} = \begin{bmatrix} -2(a_1+a_2)\rho-2a_4\rho & 0 & 0 & 0 \\ 2(a_1+a_2)\rho & -(a_1+a_2)\rho & 0 & 0 \\ 0 & -(a_3+a_4)\rho & 1-2a_3\rho & 0 \\ 2a_4\rho & (a_3+a_4)\rho & 2a_3\rho & 0 \end{bmatrix} \begin{bmatrix} P_{S1}(x) \\ P_{S2}(x) \\ P_{S3}(x) \\ P_{S4}(x) \end{bmatrix}$$

$$\begin{aligned} P_{S4a}'(x) &= 2a_4\rho P_{S1}(x) \\ P_{S4b}'(x) &= (a_3+a_4)\rho P_{S2}(x) \\ P_{S4c}'(x) &= 2a_3\rho P_{S3}(x) \end{aligned}$$

# SOLUTION TO THE SYSTEMS OF DIFFERENTIAL EQUATIONS

Solution to the preceeding system of differential equations can readily be obtained. One has for the system state probabilities provided  $a_3 \neq a_1 + a_2 + a_4$

$$P_{S1}(x) = e^{-2(a_1+a_2+a_4)\rho x}$$

$$P_{S2}(x) = \frac{2(a_1+a_2)}{a_3-a_1-a_2-a_4} \left[ e^{-2(a_1+a_2+a_4)\rho x} - e^{-(a_1+a_2+a_3+a_4)\rho x} \right]$$

$$P_{S3}(x) = \frac{(a_1+a_2)^2}{(a_3-a_1-a_2-a_4)^2} e^{-2a_3\rho x} + \frac{(a_1+a_2)^2}{(a_3-a_1-a_2-a_4)^2} e^{-2(a_1+a_2+a_4)\rho x} - \frac{2(a_1+a_2)^2}{(a_3-a_1-a_2-a_4)^2} e^{-(a_1+a_2+a_3+a_4)\rho x}$$

$$P_{S4}(x) = \frac{a_4}{a_1+a_2+a_4} \left[ 1 - e^{-2(a_1+a_2+a_4)\rho x} \right]$$

$$P_{S4b}(x) = \frac{(a_1+a_2)(a_3+a_4)}{(a_3-a_1-a_2-a_4)(a_1+a_2+a_4)} \left[ 1 - e^{-2(a_1+a_2+a_4)\rho x} \right] - \frac{(a_1+a_2)(a_3+a_4)}{(a_3-a_1-a_2-a_4)(a_1+a_2+a_3+a_4)} \left[ 1 - e^{-(a_1+a_2+a_3+a_4)\rho x} \right]$$

$$P_{S4c}(x) = \frac{(a_1+a_2)^2}{(a_3-a_1-a_2-a_4)^2} \left[ 1 - e^{-2a_3\rho x} \right] + \frac{(a_1+a_2)^2 a_3}{(a_3-a_1-a_2-a_4)(a_1+a_2+a_4)} \left[ 1 - e^{-2(a_1+a_2+a_4)\rho x} \right] - \frac{(a_1+a_2)^2 a_3}{(a_3-a_1-a_2-a_4)^2 (a_1+a_2+a_3+a_4)} \left[ 1 - e^{-(a_1+a_2+a_3+a_4)\rho x} \right]$$

$$P_{S4}(x) = 1 - P_{S1}(x) - P_{S2}(x) - P_{S3}(x) = P_{S4a}(x) + P_{S4b}(x) + P_{S4c}(x)$$

The survival probabilities for the plow and the tank are simply obtained by summing the appropriate state probabilities:

$$\text{Prob (Tank survives)} = P_{S1}(x) + P_{S2}(x) + P_{S3}(x)$$

$$\text{Prob (Tank and both plows survive)} = P_{S1}(x)$$

$$\text{Prob (Tank and at least one plow survives)} = P_{S1}(x) + P_{S2}(x)$$

In part II this model is applied to obtain a model for obtaining the optimum AM minefield fuzing mixture.

## PART II - OPTIMUM ANTI-MATERIEL MINEFIELD FUZING

### INTRODUCTION

The purpose of this section is to present an approach for determining the optimum fuzing mixture for AM minefields.

The function of employing different fuzes in a minefield is to minimize the effects of countermeasures employed by the enemy. In order to understand the advantages and limitations of the various mine types, one must examine the various strategies an enemy tank company commander can utilize. A partial list is given below.

1. No countermeasure
2. A plow can be mounted in front of each tank track to push mines aside.
3. A roller can be mounted in front of each track to roll over and thus detonate mines.
4. A line charge can be employed utilizing the shock wave it produces to detonate mines.

Let us now examine the effects of the different munitions against the tank under the countermeasure tactics listed above.

#### a. No countermeasure

If no countermeasures are employed the tank is of course vulnerable to all the mine types present in the minefield.

#### b. Plows are utilized

As long as the plows are functioning most of the PR and LI munitions will be harmlessly pushed aside. The AH mines will detonate against the plow. When a plow is destroyed the corresponding tank track becomes vulnerable to all mine types. Computation of the tank survival probability in this case requires the use of the Markov plow model developed in the previous section. Note that the AH mine type serves as a counter to plows.

#### c. Rollers are utilized

A roller placed in front of a tank track is a massive virtually indestructible object. The roller will harmlessly detonate practically all



AH and PR type munitions, and will initiate practically all LI munitions. The resulting tank mobility kill probability will depend upon its velocity and the length of the time delay incorporated in the long impulse mine: If the velocity of the tank is significantly slow or fast the mine will harmlessly detonate in front or behind the tank. Note that LI munitions serve as a counter to rollers.

d. A line charge is employed

A line charge will serve to modify the composition of minefield which the tank encounters. Most AH and some PR and LI munitions will be harmlessly detonated.

METHOD OF ANALYSIS

For each countermeasure the enemy employs one can calculate the tank survival probability as a function of the minefield parameters  $\rho x$ ,  $\lambda_1$ , and  $\lambda_2$ . Denote these survival probabilities as follows:

$P_1(\rho x, \lambda_1, \lambda_2)$ : Tank survival probability without countermeasures

$P_2(\rho x, \lambda_1, \lambda_2)$ : Tank survival probability when plows are employed

$P_3(\rho x, \lambda_1, \lambda_2)$ : Tank survival probability when rollers are employed

$P_4(\rho x, \lambda_1, \lambda_2)$ : Tank survival probability when a line charge is utilized.

Essentially, one has the situation where the enemy has available four strategies corresponding to all possible minefield mixtures. One approach to determine the optimum munition mix is to minimize a weighted average of the tank survival probabilities obtained when each countermeasure is employed separately. To this end, let

$$0 \leq b_1, b_2, b_3, b_4 \leq 1$$

$$\text{satisfying } b_1 + b_2 + b_3 + b_4 = 1$$

Form the expression

$$W = b_1 P_1(\rho x, \lambda_1, \lambda_2) + b_2 P_2(\rho x, \lambda_1, \lambda_2) + b_3 P_3(\rho x, \lambda_1, \lambda_2) + b_4 P_4(\rho x, \lambda_1, \lambda_2)$$

and choose that mixture which minimizes this weighted sum expression. It should be noted that depending on the weights chosen emphasis can be placed on a particular countermeasure. For example if  $b_1 = 1$ ,  $b_2 = b_3 = b_4 = 0$ , all emphasis is placed on the case where no countermeasures are utilized. The mixture thus obtained would be optimum if one was certain no countermeasures would be used. It is necessary for the user to define an appropriate set of weights so that results obtained are meaningful to the problem under study.

## REQUIRED INPUTS

Data input requirements are for the most part repeat of that required for the plow model in part b. Additional data is of course required to describe the cases of the roller and line charge.

### REQUIRED FOR ROLLER MODEL

#### Detonation Probabilities

Against Roller:

<u>Mine Type</u>	<u>Probability of Detonation</u>
AH	DR <sub>1</sub>
PR	DR <sub>2</sub>
LI	DR <sub>3</sub>

Against Tank Track:

<u>Mine Type</u>	<u>Probability of Detonation</u>
AH	TD <sub>1</sub>
PR	TD <sub>2</sub>
LI	TD <sub>3</sub>

#### Tank Kill Probability/Detonation

##### Detonation on Roller

<u>Mine Type</u>	<u>Probability of Tank Mobility Kill</u>
AH	0
PR	0
LI	PLI (is velocity dependent)

##### Detonation on Tank Track

<u>Mine Type</u>	<u>Probability of Tank Mobility Kill</u>
AH	TK <sub>1</sub>
PR	TK <sub>2</sub>
LI	TK <sub>3</sub>

### Required for Line Charge Model

#### Detonation Probabilities

<u>Mine Type</u>	<u>Prob mine is not detonated by line charge</u>
AH	PRC <sub>1</sub>
PR	PRC <sub>2</sub>
LI	PRC <sub>3</sub>

Fraction of minefield covered: FC (This represents the percentage of the minefield depth that can be cleared with a single line charge.)

## EXPRESSIONS FOR THE TANK SURVIVAL PROBABILITY

We now give expressions for the tank survival probability in terms of the input parameters. We list both the differential kill probability and accumulated kill probability

### 1. No countermeasures

$$P(\text{Tank mobility kill from } x \text{ to } x+\Delta x / \text{Tank alive at } x) \\ = 2K_T \Delta x \lambda_1 \rho TD_1 TK_1 + 2K_T \Delta x \lambda_2 \rho TD_2 TK_2 \\ + 2K_T \Delta x (1-\lambda_1-\lambda_2) \rho TD_3 TK_3$$

$$P_1(\rho x, \lambda_1, \lambda_2) = e^{-e_1 \rho x}$$

where

$$e_1 = 2K_T \lambda_1 TD_1 TK_1 + 2K_T \lambda_2 TD_2 TK_2 + 2K_T (1-\lambda_1-\lambda_2) TD_3 TK_3$$

### 2. Mine Clearing Plows are employed

Expressions for the tank survival probability  $P_2(\rho x, \lambda_1, \lambda_2)$  are developed in Part I.

### 3. Rollers are employed

If rollers are employed as a countermeasure, a tank mobility kill in  $x$  to  $x+\Delta x$  can occur in one of two ways

i. A mine type may fail to detonate on the roller, detonate on the tank track and inflict a tank mobility kill

ii. A LI munition may be initiated by the roller, detonate on the tank track, and thereby cause a tank mobility kill.

One obtains for the differential tank mobility kill probability

$$P(\text{Tank mobility kill from } x \text{ to } x + \Delta x / \text{Tank alive at } x)$$

$$\begin{aligned} &= 2K_T \Delta x \lambda_1 \rho (1-DR_1) TD_1 TK_1 \\ &\quad + 2K_T \Delta x \lambda_2 \rho (1-DR_2) TD_2 TK_2 \\ &\quad + 2K_T \Delta x (1-\lambda_1-\lambda_2) \rho (1-DR_3) TD_3 TK_3 \\ &\quad + 2K_T \Delta x (1-\lambda_1-\lambda_2) \rho DR_3 PLI \end{aligned} \quad \left. \begin{array}{l} \text{way (i)} \\ \text{way (ii)} \end{array} \right\}$$

Therefore

$$P_3(\rho x, \lambda_1, \lambda_2) = e^{-e_3 \rho x}$$

$$\begin{aligned} \text{where } e_3 &= 2K_T (1-DR_1) \lambda_1 TD_1 TK_1 + 2K_T (1-DR_2) \lambda_2 TD_2 TK_2 \\ &\quad + 2K_T (1-DR_3) (1-\lambda_1-\lambda_2) TD_3 TK_3 \\ &\quad + 2K_T DR_3 (1-\lambda_1-\lambda_2) PLI \end{aligned}$$

#### 4. A line charge is used

##### i. When the entire minefield is covered:

The case where a line charge is employed is similar to no countermeasure case except that the minefield density and mixture is modified. The differential kill probability is given by

$$\begin{aligned}
 &P(\text{Tank mobility kill in } x \text{ to } x+\Delta x/\text{Tank alive at } x) \\
 &= 2K_T \Delta x \lambda_1 \rho \text{PRC}_1 \text{TD}_1 \text{TK}_1 + 2K_T \Delta x \lambda_2 \rho \text{PRC}_2 \text{TD}_2 \text{TK}_2 \\
 &\quad + 2K_T \Delta x (1-\lambda_1-\lambda_2) \rho \text{PRC}_3 \text{TD}_3 \text{TK}_3
 \end{aligned}$$

Therefore

$$P_4(\rho x, \lambda_1, \lambda_2) = e^{-e_4 \rho x}$$

where

$$\begin{aligned}
 e_4 &= 2K_T \lambda_1 \text{PRC}_1 \text{TD}_1 \text{TK}_1 + 2K_T \lambda_2 \text{PRC}_2 \text{TD}_2 \text{TK}_2 \\
 &\quad + 2K_T (1-\lambda_1-\lambda_2) \text{PRC}_3 \text{TD}_3 \text{TK}_3
 \end{aligned}$$

ii. When a portion FC, of the minefield is covered by the line charge one has

$$P_4(\rho x, \lambda_1, \lambda_2) = e^{-e_4 \text{FC} \rho x} \cdot e^{-e_1 (1-\text{FC}) \rho x}$$

#### A NUMERICAL EXAMPLE

Let us conclude this section with the following numerical example.

Weights:

No countermeasure	$b_1 = 0.0$
plow	$b_2 = 0.5$
roller	$b_3 = 0.12$
line charge	$b_4 = 0.38$

Minefield density: 1.0 mines/ft  
 Fraction coverage of line charge: 0.5

The results are given in figures 2 and 3. Figure 2 is a contour plot giving equi-survivability contours as a function of minefield composition. Note from this figure that the tank survival probability for specified parameters is very insensitive to the mixture employed. For example, if one chooses a mixture of 40% LI one would obtain a tank survival probability of .44 which differs from the optimum by only .0415 or about 10%. Figure 3

is a sensitivity plot depicting the dependence of the optimum fuzing mixture on the linear minefield density. The figure clearly demonstrates that the optimum mixture is a very sensitive function of the minefield density.

#### CONCLUSIONS

In conclusion, it should be noted that the Markov approach represents an elementary solution to a very important problem area in mine systems development. Hopefully, this represents only a beginning and that the analytical and engineering tools that comprise the body of knowledge referred to as Operations Research will yield numerous powerful analytical methods to aid in the solution of mine and other weapon system design and effectiveness problems.

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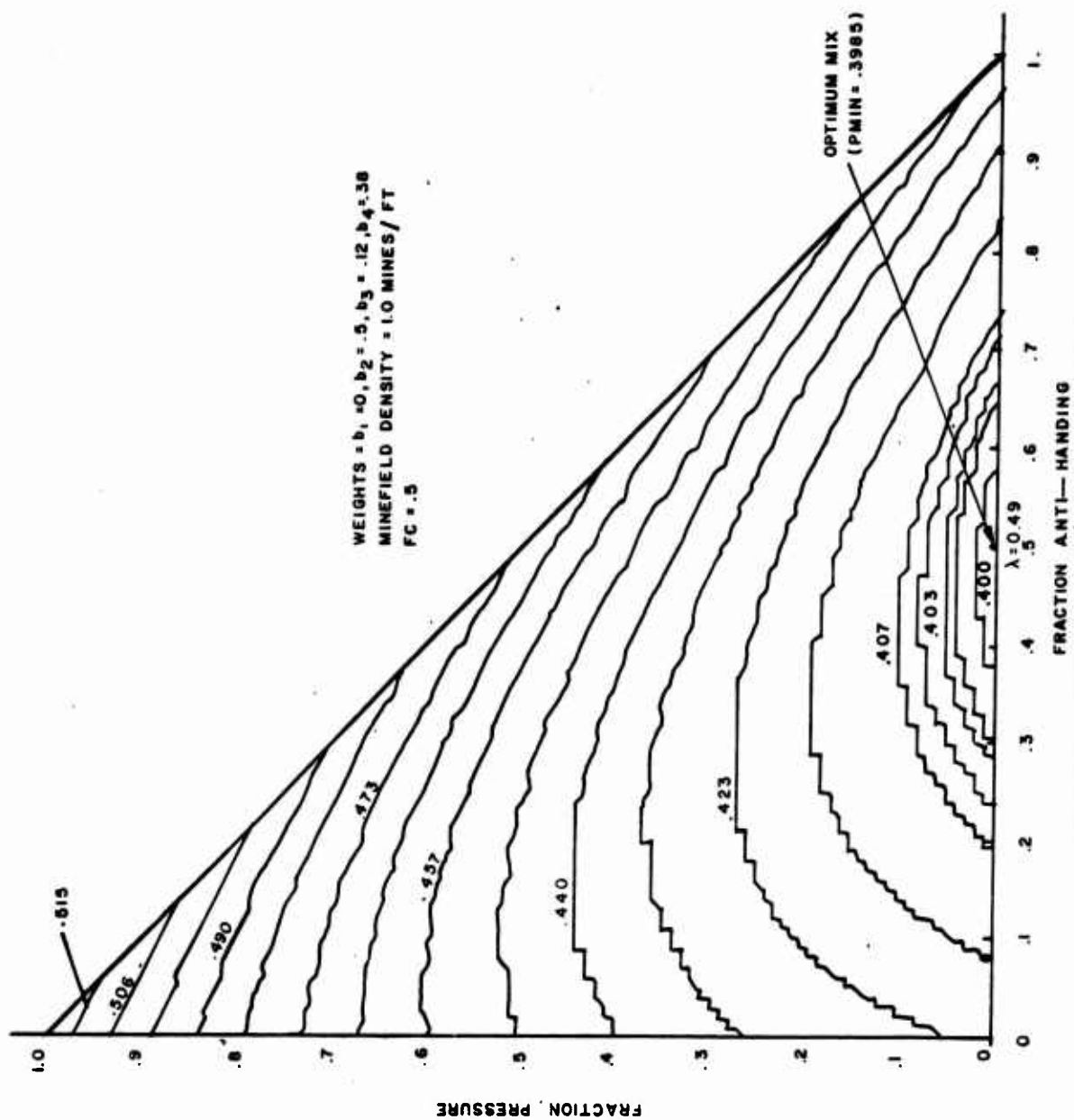


FIGURE 2 TANK SURVIVAL PROBABILITY CONTOURS



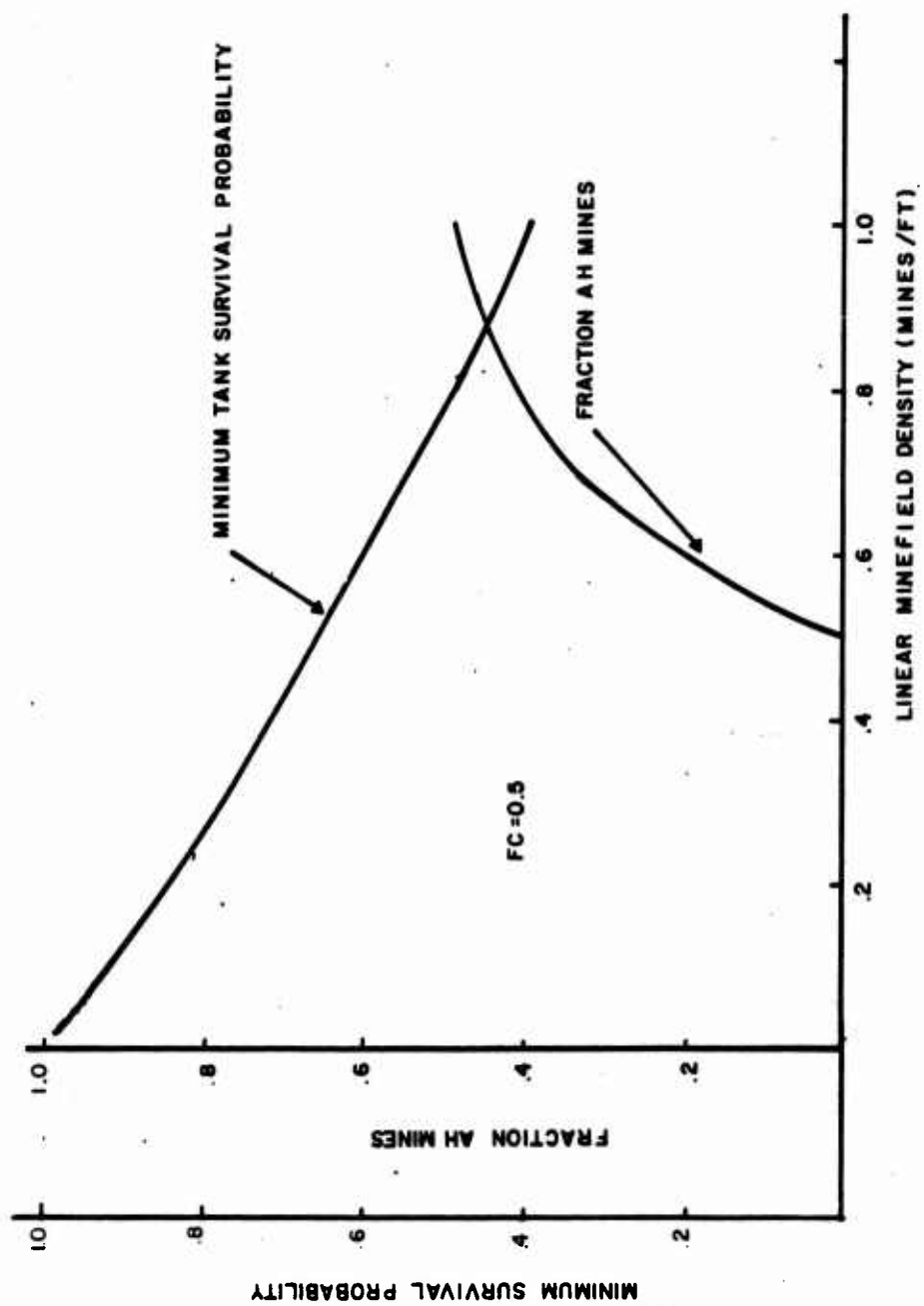


FIG. 3 DEPENDENCE OF OPTIMUM MIX ON MINEFIELD DENSITY